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Human's Digital Space – What about the Metrics?

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1 ABSTRACT

At the recent conference, CORP 2016, the idea about free space around a human has been discussed. In GIS systems, there are 1D, 2D, 3D, 3D+t representations of geographical elements and others features. According to GIS applications, a human also can be regarded as an object of GIS presentation and investigation for end users, software agents etc.

In this paper, we would like to continue our research regarding human's digital space (HDS). A very important object of our discussion is a metric for HDS properties. However, if we are speaking about metrics, we should take a look at classical mathematics. A short review of mathematical definitions of space and their metrics is also discussed. Our preliminary research has shown that Hausdorff measure (a special mathematical measure used to calculate length, area and volume of non-specific figures) discovers new opportunities for realisation of different kinds of business logics in complex multidimensional and implicitly specified spaces. In HDS, investigation the Hausdorff measure could be used as an approach for numerical interpretation of HDS properties.

Keywords: Human's Digital Space, Hausdorff measure, GIS, 3D, Space definition

2 INTRODUCTION

In philosophy and some classical mathematics sources, we can find that the metaphysician Immanuel Kant said that the concepts of space and time are not empirical ones derived from experiences of the outside world—they are elements of an already given systematic framework that humans possess and use to structure all experiences. Kant referred to the experience of "space" in his Critique of Pure Reason [5] as being a subjective "pure a priori form of intuition". According to Kant, knowledge about space is synthetic, in that statements about space are not simply true by virtue of the meaning of the words in the statement. According to this, Kant rejected the view that space must be either a substance or relation. Instead, he came to the conclusion that space and time are not discovered by humans to be objective features of the world, but imposed by us as part of a framework for organizing experience.

Modern scientists usually fall under two opposing groups: overt opponents and adherers of Kant's idea. While not being experts in a field of philosophical inquiry, we, however, can note that we are closer to Kant's idea, since in computer science, and especially in geoinformatics, this idea becomes a very good ideological basis for designing of actual technology and applications [8].

For a common case, space exists independently of events and experiments. Taking in mind philosophical aspects of space, we should also note that the GIS has strong applied sense. According to this, two ideas should be investigated together: absolute space and relative space [2].

Considering such complex history of this term and its current ambiguity, we should address its mathematical definition. All the more so because in our field of study, in geoinformatic field, anthropological nuances do not aid, but, in fact, obscure this truly important matter. In [1] and other mathematical sources, the mathematical term of "space" can be defined as [1] "mathematical set that possesses structure defined by axiomatic properties of its elements (e.g. points in geometry, vectors in linear algebra, events in probability theory and etc.). Subset of space is called "subspace" if space structure initialises the structure of same type on this subset (the exact definition depends on space type)".

The term "space" for mathematics turned out to be extremely useful. Let us give a partial list of various types of space in mathematics:

- Affine space is a space that generalizes the properties of Euclidean spaces. It is mostly similar to vector space, however, affine space is distinctive by the fact that all its points are equal (in particular, the concept of zero point is not defined in it).
- Banach space is a complete normed vector space.

- Probability space is a concept, introduced by A.N. Kolmogorov in 1930s in order to formalise the concept of probability that originated the rapid development of probability theory as a strict mathematical discipline.
- Hilbert space is a generalisation of Euclidean space that allows infinite number of dimensions.
- Euclidean space, in initial terms, is a space which properties are described by Euclidean geometry axioms. In this case, it is assumed that the space is three-dimensional. In modern understanding, in more general sense, it can denote one of closely related objects: finite-dimensional real vector space with positively defined scalar product, or metric space corresponding to such vector space.
- Normed space is a vector space on which a norm is defined.
- Vector space is a mathematical structure that represents a set of elements (points) called vectors, for which operations of addition and multiplication by number (scalar). These operations are defined by eight axioms.
- Metric space is a set for which distance with certain properties (metric) between any pair of elements is defined.
- Minkowski space is a four dimensional pseudo-Euclidean space with signature, suggested as an interpretation of space-time in the special theory of relativity. For every event there is a corresponding point in Minkowski space in Galilean coordinates.
- Topological space is a set with additional structure of certain type (so called topology). It is one of objects of study for one of branches of mathematics also called topology. Historically, the term "topological space" originated as generalisation of metric space.

The classification of mathematical term "space" given above has formed historically and it reflects the level of generalisation of fundamental concepts of "point", "measure" and some other qualities. A question emerges: can HDS belongs to one of the listed types or to a number of them? Of course, it can. For example, common for ordinary person Euclidean space, topological space (navigational format C57) and some others.

At the same time, we should note that at present time such traditional notions like "space", "set", "point" are failing to satisfy not only theorists but practitioners as well. These contradictions can be clearly represented for HDS. The notion of "point" is so abstract that it becomes more and more difficult to create applicable along with theoretical interpretation and to refer it to one of the types of mathematical spaces, described above. The case is that it is physically impossible to denote the term "point" in traditional sense (some abstraction indicating coordinates). We always deal with a certain multidimensional, at least with dimension of three, neighbourhood of some point that cannot always be denoted as a centre of some local coordinate system. In addition, even if we do so, there is no positive gain from this abstraction. An attempt to solve practical tasks for HDS as a GIS object leads to piling of large abstractions and relations between them. It is no coincidence that in Java programming language there are no simple abstract data types like point, line etc., as there were in preceding languages. They used to be practically identical to notion of "point" in Euclidean space. In Java we initially have a notion of "object" and it is very right. In other words, we have a defined set with given structure. Term "object" cannot be referred to the term "set" in algebra. This is a new concept more closely related to the concept of "category" [7]. Therefore, the term "point" cannot be constricted to just some coordinate vector. Moreover, consequently, the term cannot be referred to any of the known abstract data types apart from "object". In this context, we have no simple analogue for known mathematical definitions of space.

At this rate, the question arises: what is space for HDS in GIS? If it is a point, the basis of practically any space, then, after generalisation of "point" concept, we have to define what space for GIS denotes. Complexity of analysis of this concept also arises from the fact that GIS is at the same time a theory, a technology and a practice. At that, theory, technology and practical application are very closely connected and often change places in time in unnatural order (compared to traditional concept of fundamental science, application-oriented technologies and practice). In computer sciences, it has been long noted that application-oriented researches and technologies frequently outpace theoretical, fundamental. We can assume that HDS is defined by such formats or sets of numbers that are needed for business analytics realisation (for user's convenience). Alternatively, this space is defined by such space (usually a mathematical concept) that represents GIS business analytics. In scientific work [3] it is shown that GIS is specified on multidimensional space. At that, all mathematical space paradigms turn out to be either trivial or useless due to interpretation complexity. However, we should clearly understand what we are dealing with, what interpretation capabilities we possess. Without formal definition of HDS for GIS it is unlikely to succeed in designing and application. Space is a foundation of all model system, specified on this space. Models are building blocks for application tasks and business logic that created for simplified representation, study and analysis of objective and/or abstract reality.

3 SPACE DEFINITION

Having determined that such categories as “time” and “space” are key concepts for HDS, we still have to give them a definition. Concept of time is universal nearly for all fields of study and is unlikely to have any particularities. Exclusively, we can note that there can be several relative time scales. For example, while modelling, we can artificially slow down or speed up time, do time jumps forwards or backwards along the time line. Space is different story. Considering the fact that, as stated above, we have at least six types of major spaces (scientific fields) but, in truth, there are much more of them, we needed to regard them separately.

As was noted in introduction, any recent graduate and even user understands that most simple and at the same time the most general abstraction is a point. On its own, a point is a primitive concept with sufficiently vast set of properties. Yet, the concept of point is not independent. Without definition of space, the concept of point is meaningless and vice versa. Considering the specificity of our field of study: HDS, the point is not simply a mathematical notion but, firstly, it is a coordinate and not only a coordinate. Depending on context, the point has a whole set of properties and sometimes methods (functions).

Let us regard the concept of point for various situations form the point of view of mundane consciousness:

(1) One-dimensional case (point). The point has one coordinate and a number of other parameters.

(2) Two-dimensional case (Euclidean space). The point has two coordinates and a number of other parameters.

(3) Three-dimensional case. The point has tree coordinates and a number of other parameters.

(4) Multidimensional case. The point has a number of coordinates plus a number of other parameters.

(5) All cases above plus time. The point has time parameter added.

The point is an initial concept from which all other abstracts found for HDS can be formed, e.i. they are derivative. On the other hand, abstract concept of point should differ. Regardless of the fact that we apply HDS for specific purposes and for one, in particular case, subject area, these spaces should not overlap, else we will obtain whole system of contradictions.

Let us make one small remark. We cannot build HDS for GIS space system using axiomatic approach [3]. It means that we cannot formulate universal set of constraints and assumptions for all HDS systems. It is more than likely that we should use evolutionary approach and, perhaps, in the future it will be possible to design an axiomatic system (theory).

We have a very complex combination of spaces, rather closely interrelated, that, however, have a principal difference in fundamental concept of “space point”. Shortly, these abstract points can be defined as measurement (signal, connection, etc.), object (physical or abstract), tactical situation, threat, resource and solution. For every space measure should be defined as a mean of specifying analytics on space in favour of practical and/or abstract task solving.

4 MEASURE DEFINITION FOR SPACE

In philosophical sense, measure is a philosophical category denoting unity of qualitative and quantitative qualities of some object. According to A.P. Ogurtsov [10], this category generalises means and results of measuring objects. Measure analysis derives from importance of variation intervals of quantitative values, in terms of which we can talk of object’s quality preservation. Measure category is closely related to a number of philosophic notions along with those falling into fields of ethics and aesthetics. In mathematics, measure is a common name for different types of generalisation of notions of Euclidean length, area and n-dimensional volume. There are various specifications to the notion of measure:

- Jordan measure is an example of finitely additive measure or one or the ways of formalising notions of length, area and n-dimensional volume in Euclidean space.
- Lebesgue measure is an example of denumerable additive measure, is a continuation of Jordan measure on more vast class of sets.
- Riemann measure (Riemann integral) is an area of region under a curve (a figure between graph of function and abscissa).
- Hausdorff measure is a special mathematical measure. Necessity of introduction of such measure derived from the need to calculate length, area and volume of nonspecific figures that can be not specified analytically. Application of such generalisation for HDS discovers new opportunities for realisation of different kinds of business logics in complex multidimensional and implicitly specified spaces. It is nearly impossible to do analytically, or, at least, very difficult. Level of difficulty is common for the task of calculating the volume of available networks [3G, 4G, Wi-Fi and etc.] outdoors for particular user, or for given type of users, taking into account complex surrounding space (buildings, metal fences and etc.). Numerical values of such space can be calculated directly in the process of field task solving (by specifying Hausdorff measure and step-by-step calculation), or using imitation modelling method when field is already given.

5 HAUSDORFF MEASURE FOR HDS ESTIMATION

In ordinary life, many things are suitable for our comprehension. Very typical parameters like length, area, volume etc. require no additional study for an individual. What parameter can be used as an analog of typical parameters in HDW? Are there similar analogs for HDS?

In the previous paper [10] we have noted that the stated similarity of the mathematical form of individual profile and track representation as a vector of values of a certain set of characteristics allows us to combine the methods of individual "track" identification and individual "profile" identification.

In other words, if we have definitions of "track" and "profile" of individual, they are subjects of estimation for HDS. However, there are no typical abstractions like those that we have in IGIS subject domain. One way to approach such atypical abstractions is to apply the Hasdorff measure.

For example, let us regard an Euclidean plane with Cartesian coordinates. We shall divide it into small squares of side ε > 0 using lines, parallel to coordinate axes. Let us establish a bounded set S on this plane. N(ε) is a minimal number of squares that cover S. If S is some known figure, e.g. a sphere, then its area is given by:

$$S = \lim_{\varepsilon \to 0} \varepsilon^2 N(\varepsilon)$$

It can be said that when ε→0 the number of covered squares N(ε) grows like S/ε2 . The denominator of this fraction indicates dimensionality (it equals 2), and the numerator indicates area's size, or, figuratively speaking, size of 2-measure.

In a common case, we will assume that the set S has dimension d = dimS, 0 <= d <= 2, if, while ε →0, the number of cells N(ε) grows like C/εd , where C is some positive constant called d-measure of the set S. It means that:

$$C = \lim_{\varepsilon \to 0} \varepsilon^d N(\varepsilon)$$

One of the simplest examples is for the plane. In case of 3-dimensional physical representation, e.g. radiation fields, we are given a set of spheres with a radius ε. This case is common for estimation of availability of various kinds networks for an individual in different geographical coordinates and conditions: apartment, office, cafeterias, street, subway, car, plane, yacht etc. All this is normally conceived by human consciousness. However, if we begin to contemplate the topology of computer networks while trying to analyze tracks of data transmission to or from us from one or many sources, situation goes beyond our traditional comprehension. In this case we can derive Hausdorff measure with fractal dimension, e.g. d can have various values, which can be difficult to imagine or decipher. This poses the question: can HDW be interpreted as an element of global network, of global DW, with help of one or several measures? Can we correctly create a business logic for estimation of vulnerability and protection of HDW that is an element of global network (DW)? These are only a couple of questions that arise as we delve into the subject of HDW. If we cannot imagine spaces like DW, HDW as measurable, then there is no sense in speaking about the degree of protection of modern individual, of his rights and freedoms.

6 CONCLUSION

Definition of the term "space" as an independent object of study in IGIS opens vast perspectives for study of nonspecific objects that can not be represented in the familiar for a human form: in form of a point, line, sphere, polyline or their combinations. Practically all geographical formats were developed with purpose of creating the simplest set of abstractions, from which all other object in GIS are derived. In mathematical terms, these are understandable metrical spaces and functions, defined on them.

Similar situation was in mathematics until Hausdorff noted the narrowness of such representation. Many mathematicians have noticed before those natural phenomena like shoreline, snow, clouds etc. can not be represented as some aggregate of traditional notions like a point, line etc. and can not be analytically described.

Same situation we now have in IGIS, when efforts are taken to integrate IGIS into complex information systems and all more so when global information systems are based on IGIS technologies.

The more complex structure have objects that are connected to modern human activities. This problem was discussed on the previous CORP conference. Understanding the fact that any modern human resides in global informational space, the question has risen: what belongs solely to him and how can he estimate, control and protect this "property"? Personal space can not be labeled as simple "property" category given that it integrates such notions like freedom, independence, personal life etc.

Estimation of individual's personal space is not a banal thing and should have a clear and comprehensible for each individual. Application of Hausdorff measure and its further development will allow to approach the estimation and understanding of place an role of "personal" HDS in the hierarchy of complex subspaces of DW. Each human has the right to understand their rights, freedoms and responsibilities in the global DW.

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